

## “An AQL System for Lot-By-Lot Acceptance Sampling By Attributes Selecting an O.C.Curve”

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**Abstract:** The choice among various possible types of acceptance inspection procedures is essentially an economic one. In making a decision regarding acceptance inspection for any particular purpose, it may be desirable to consider not only various possible systems or procedures of acceptance sampling by attributes but also the alternatives of (1) no inspection at all, but imposition of a requirement that statistical evidence of satisfactory quality be provided with each lot; (2) 100% inspection; and (3) possibilities of acceptance sampling by variables. It also is true that a satisfactory evaluation of all the pertinent economic factors is often quite difficult. For this reason, the choice of an acceptance procedure is commonly made on an intuitive basis. An important element of the selection of an acceptance inspection procedure should be the probable contribution of the procedure to quality improvement. The acceptance sampling systems and procedures described in this and the following chapters have often been strikingly successful in leading to such improvements.

**Key Words:** AQL, MIL-STD-105D, AOQL, Acceptance Sampling.

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### I. Introduction and Review of Related Literature:

It is pointed that three points on the OC curve have been used as methods of indexing sets of sampling plans. These, and other statistical measures of performance, provide the logical basis for combining sets of plans into sampling systems. For example, certain Dodge-Romig tables employ the  $P_{0.10}$  point as the index to two sets of tables. The point  $100 p_{0.10}$  is termed the Lot Tolerance Percent Defective (LTPD), the percent defective that has a probability of acceptance of 0.10. Thus the two sets of tables are indexed on Consumer's Risk  $\beta$ , where  $\beta$  equals 0.10. Other procedures employ the point-of control or indifference quality  $p_{0.50}$  as the index base. The system of sampling plans most frequently used in the United States is that of the ABC standard, also known as MIL-STD-105 and ANSI/ASQC Standard Z1.4. That system is indexed on the AQL (acceptable quality level). While not an exact point on the OC curve in terms of Producer Risk, lots submitted at the AQL have a risk of rejection  $\alpha$  in the range 0.01 to 0.10. Selected points on the OC curve need not be the only indexes of sampling plans. Another such index is the Average Outgoing Quality Limit (AOQL). Two of the four sets of Dodge-Romig tables described in literature are indexed on the AOQL. All sampling plans in the Dodge-Romig tables aim at minimizing the average total inspection (ATI) considering both sampling inspection and screening inspection of rejected lots.

The AQL concept was first devised in connection with the development of statistical acceptance sampling for the Ordnance Department of the U.S. Army. The Ordnance tables and procedures were developed in 1942 by a group under the direction of distinguished engineers from the Bell Telephone Laboratories. With some changes and extensions, these became the Army Service Forces tables developed by the same group. These tables permitted single and double sampling, with double sampling preferred wherever practicable. Statistical sampling tables and procedures developed for the Navy by the Statistical Research Group of Columbia University were first issued in 1945. The general pattern of these tables and procedures was similar to that used by the Army Service Forces. However, multiple sampling schemes were made available, and there were other important points of difference. After the unification of the armed services, these Navy tables were adopted by the Department of Defense early in 1949 as JAN (Joint Army Navy) Standard 105. The tables were made available for public use through the publication of the SRG's volume "Sampling Inspection". MIL-STD-105A superseded JAN-STD-105 in 1950. Although the underlying pattern was similar to the preceding standards, there were again many important changes in detail. Only minor changes from 105A were involved in MIL-STD105B, adopted by the U.S. Department of Defense in 1958, and in MIL-STD105C, adopted in 1961. Revision E of MIL-STD-105 was issued by the U.S. Department of Defense in 1989. While there were many changes in the ordering of the procedures, only a few substantive changes were made, and no changes were made to the tables developed for the ABC standard and carried over into 105D and Z1.4. The substantive changes are described at appropriate points in the discussion. In order to stress the importance of this standard and its international character, it is referred to throughout this chapter as the ABC standard (from the name of the working group that prepared it). It is not merely the use of AQL systems in purchases by governmental organizations that makes such systems important. Systems based on the AQL have been widely adopted by private

industry for acceptance sampling of all kinds of products. In most cases, these systems were adapted from one of the military systems.

## II. Mathematical Model:

The probability of observing exactly  $d$  defectives is;

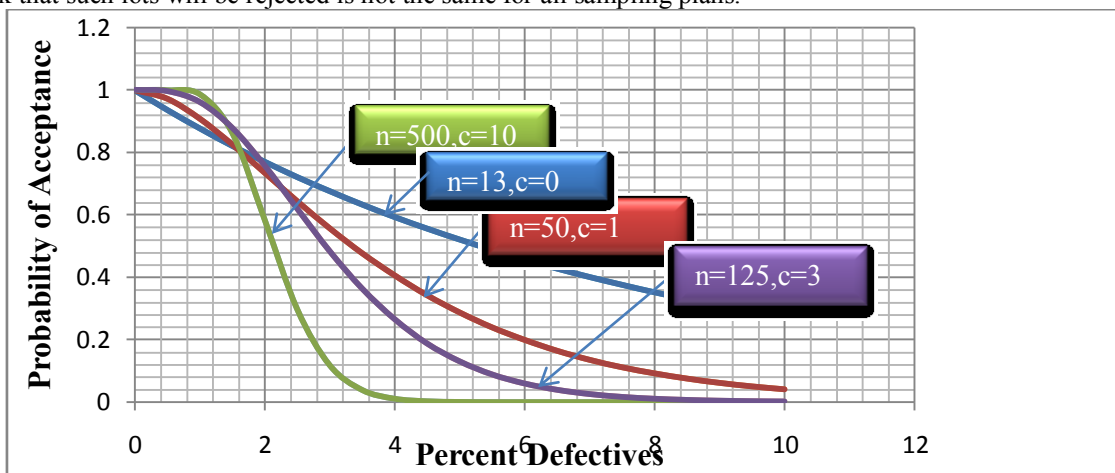
$$P\{\text{Defectives}\} = f\{d\} = \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

The probability of acceptance is just the probability that  $d$  is less than or equal to  $c$ .

$$P(a) = P\{d \leq c\} = \sum_{d=0}^c \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

### Probabilities of Acceptance of Lots Having AQL Percent Defective:

In all AQL systems, the acceptance criteria under normal inspection have been chosen to protect the producer against rejection of lots meeting the quality standard. However; in most AQL systems the Producer's Risk that such lots will be rejected is not the same for all sampling plans.



**Figure 1. OC Curve for four single Sampling plan from ABC Standard all with AQL of 0.1%**

Figure 1. Shows a portion of the OC Curves for four single sampling plans from the ABC standard all having an AQL of 1 %. It is evident that the smaller the sample size, the greater the risk the producer takes that a lot will be rejected when it is exactly 1% defective. Of course, the Consumer's Risk of accepting a lot much worse than the AQL is also much greater with a small sample size. For example, Fig. 1 shows that a 5% defective lot has more than a 0.5 probability of acceptance with  $n = 13$  and  $c = 0$ . In contrast, such a lot is practically certain to be rejected with  $n = 500$  and  $c = 10$ . It has been already noted that, when the acceptance number is zero, the OC curve has no point of inflection; it is concave upward through its entire length. If a plan with  $c = 0$  were selected to reduce the Producer's Risk of rejection of a 1% defective lot to, say, 0.05, the consumer would have even less protection against accepting bad product than is present with  $n = 13$  and  $c = 0$ . (A  $P_a$  of 0.95 for 1 % defective product with  $c = 0$  would require  $n = 5$ . With this  $n$ , even a 12% defective lot would have a better than even chance of being accepted.) In designing an AQL system, it is reasonable to give some weight to the entire OC curve of each of the sampling plans in the system rather than merely to a point near one end of the OC curve, namely, the  $P_a$  at the AQL value. In the ABC system, the probability of acceptance at the AQL value in normal inspection varies from about 0.88 for plans with the smaller sample sizes where  $c = 0$  to about 0.99 for the large sample sizes and acceptance numbers. H. F. Dodge proposed an AQL system, that eliminates the use of  $c = 0$  in single sampling under normal inspection (although  $c = 0$  is used in tightened inspection). This elimination makes practicable a standardized aimed at  $P_a$  of 0.95 at the AQL value under normal inspection.

### Selecting a Sampling Plan for Normal Inspection:

Assume that an AQL of 1.5% has been specified for a certain class of defects. Assume normal inspection with a lot size of 1,000 and inspection level II. Table K indicates that the sample size code letter is J. Table L gives acceptance criteria with normal inspection, single sampling. It tells that for code letter J and a 1.5% AQL, sample size is 80 and the acceptance number is 3. The rejection number is stated as 4. In all these single sampling plans, the rejection number is one more than the acceptance number. Table 1 gives acceptance

criteria with normal inspection, double sampling and gives criteria with normal inspection, multiple sampling. For code letter J and an AQL of 1.5%, these are:

Sample number	Sample Size	Cumulative Sample Size	Acceptance Number	Rejection Number
Double:				
First	50	50	1	4
Second	50	100	4	5
Multiple:				
First	20	20	t	3
Second	20	40	0	3
Third	20	60	1	4
Fourth	20	80	2	5
Fifth	20	100	3	6
Sixth	20	120	4	6
Seventh	20	140	6	7

**Table 1: Acceptance Not Permitted At This Sample Size.**

The standard gives OC curves for all single sampling plans for normal and tightened inspection. It is stated that “curves for double and multiple sampling are matched as closely as practicable.” The various issues involved in choosing among single, double, and multiple sampling are discussed in this paper. For the discussion of a number of aspects of the standard, there should be concentration on a single sampling. In all the military AQL systems using sample size code letters, the code letter in tightened inspection is determined just as in normal inspection. The acceptance criteria under tightened inspection in the ABC standard are shown in Tables M, P, and S. The relationship between criteria under normal and tightened inspection may be illustrated with reference to the normal plans for a 1 % AQL that were shown in Fig 1.

Code Letter	Normal		Tightened	
	n	c	n	c
E	13	0	20	0
H	50	1	80	1
K	125	3	125	2
N	500	10	500	8

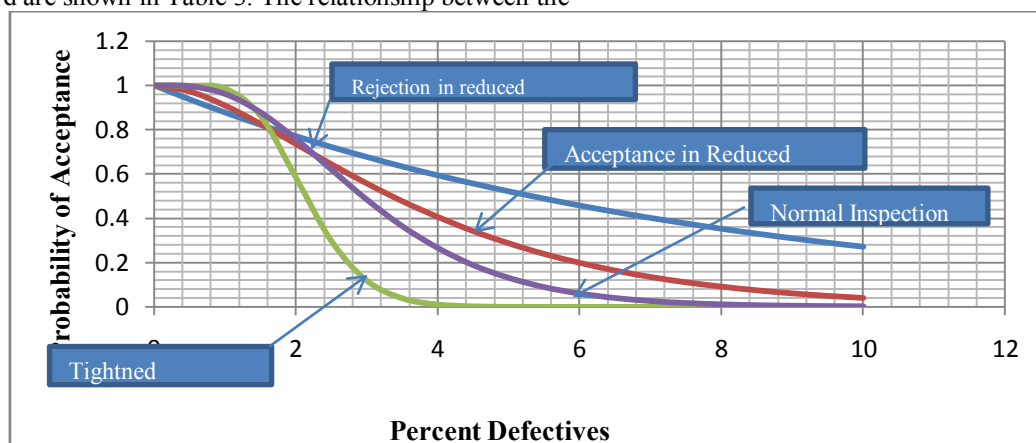
**Table 2: Acceptance Criteria under Tightened Inspection in the ABC Standard**

**Acceptance Criteria under Tightened Inspection:**

In the first three cases the criteria under tightened inspection for a 1 % AQL are the same as the criteria under normal inspection for the stated code letter for the next lower AQL, namely, 0.65%. In most cases in the ABC standard, the tightened criteria are identical with the normal criteria for the next lower AQL class, although there are a number of exceptions. Later in this paper there is an illustration of the difference between the OC curves in normal and tightened inspection. The various AQL systems have differed considerably in this respect.

**Acceptance Criteria under Reduced Inspection:**

In all the military AQL systems using sample size code letters, the code letter in reduced inspection has been determined just as in normal inspection. The acceptance criteria under reduced inspection in the ABC standard are shown in Table 3. The relationship between the



**Figure 2: OC Curves under Normal Tightened and Reduced Inspection Single Sampling.**

criteria under normal and reduced inspection may be illustrated with reference to the single sampling normal plans for a 1 % AQL that were shown in Fig. 1.

Code Letter	Normal		Reduced	
	n	Accept	Reject	n
E	13	0	1	5
H	50	1	2	20
K	125	3	4	50
N	500	10	11	200

**Table 3: Acceptance Criteria under Reduced Inspection in the ABC Standard**

The reader will observe that the acceptance criteria shown for code letters H, K, and N under reduced inspection all have an area of indecision in which the lot is neither accepted nor rejected. The standard states that, whenever the number of defectives falls in this indecision region (for example, if there should be exactly 1 defective in the sample of 20 with code letter H), the lot in question shall be accepted but reduced inspection shall be discontinued and normal inspection reinstated.

### III. Conclusions:

A provision for reduced inspection is not a necessary part of an acceptance/rejection plan. Nevertheless, such a provision is based on a principle that is economically sound. This principle is to concentrate inspection attention on those products and quality characteristics where the quality history is doubtful and to give less attention where the quality history is very good.

The consumer's savings in inspection costs under reduced inspection are apparent. The producer's advantages are not quite so obvious. However, because the acceptance criteria in reduced inspection are not so stringent, the producer receives added protection against lot rejection. The producer may also have a real sense of accomplishment in having qualified for reduced inspection. Hence, from the consumer's viewpoint, the provisions for reduced inspection in any acceptance program may provide a useful non financial incentive to the producer to improve quality. This point is illustrated in Example 14.1 with reference to reduced inspection in multilevel continuous sampling. Figure 1.2 shows an abbreviated flow diagram of the operation of the switching rules in the ABC standard.

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