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Structural Dynamic Reanalysis of Beam Elements Using Regression Method

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Abstract : *This paper concerns with the reanalysis of Structural modification of a beam element based on natural frequencies using polynomial regression method. This method deals with the characteristics of frequency of a vibrating system and the procedures that are available for the modification of physical parameters of vibrating structural system. The method is applied on a simple cantilever beam structure and T-structure for approximate structural dynamic reanalysis. Results obtained from the assumed conditions of the problem indicates the high quality approximation of natural frequencies using finite element method and regression method.*
Keywords: *frequency, mass matrix, physical parameters, stiffness matrix, regression method.*

I. INTRODUCTION

Structural modification is usually having a technique to analyze the changes in the physical parameters of a structural system on its dynamic characteristics. The physical parameters of a structural system are related to the dynamic characteristics like mass, stiffness and damping properities.. for a spring- mass system, mass and stiffness quantities are the physical properties for the elements. The parameters for a practical system such as a cantilever beam and T-structure may be breadth, depth and length of a beam element. The changes in the parameters will effect the dynamic characteristics i.e., both mass and stiffness properties of the beam . [1]

Reanalysis methods are intented to analyzeeffectively about the beam element structures that has been modified due to changes in the design (or) while designing new structural elements. The source information may be utilized for the new designs. One of the many advantages of the elemental structure technique is, having the possibility of repeating the analysis for one (or) more of the elements making the use of the work done by the others. This will gives the most significant time saving when modifications are requried.[2]

Development of structural modification techniques which are them selves based on the previous analysis. The modified matrices of the beam element structures are obtained , with little extra calculation time, can be very easy and useful. The General structural modification techniques are very useful in solving medium size structural problems as well as for the design of large structures also.

The main object is to evaluate the dynamic characteristics for such changes without solving the total (or) complete set of modified equations.

II. Finite Element Approach

Initially the total structure of the beam is divided into small elements using successive levels of divisions. In finite element analysis more number of elements will give more accurate results especially of the higher modes. The analysis of stiffness and mass matrix are performed for each element separately and then globalized into a single matrix for the total system.

The generalized equations for the free vibration of the undamped system, is[3]

$$[M]\ddot{X}+[B]\dot{X}+[K]x=f \qquad (1) \longrightarrow$$

Where M,B= αM+βK and K are the mass, damping and stiffness matrices respectively. $\ddot{X}$,$\dot{X}$and X are acceleration, velocity, displacement vectors of the structural points and "f" is force vector. Undamped homogeneous system of equation

$$M\ddot{X}+Kx=0 \qquad (2) \longrightarrow$$

Provides the Eigen value problem $[K-\lambda M]\,\phi = 0 \longrightarrow (3)$

Such a system has natural frequencies

$$\left.\begin{aligned} \lambda &= \begin{bmatrix} w_1^2 & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & w_n^2 \end{bmatrix} \\ \phi &= [\phi 1, \phi 2 \ldots\ldots \phi n] \end{aligned}\right\} \longrightarrow (4)$$

Condition:

Must satisfy the ortho normal conditions

$$\left.\begin{aligned} \boldsymbol{\phi}^T \mathbf{M}\, \boldsymbol{\phi} &= \mathbf{I}, \\ \boldsymbol{\phi}^T \mathbf{K}\, \boldsymbol{\phi} &= \lambda, \\ \boldsymbol{\phi}^T \mathbf{C}\, \boldsymbol{\phi} &= \alpha \mathbf{I} + \beta\lambda = \xi, \end{aligned}\right\} \longrightarrow \quad (5)$$

It is important note, that the matrices,

$$\widehat{M} = \phi^T \mathrm{M}\, \phi, \quad \widehat{C} = \phi^T \mathrm{C} \phi, \quad \widehat{K} = \phi^T \mathrm{K} \phi$$

Are not usually diagonalised by the eigenvectors of the original structure [4]

The stiffness and mass matrix of a beam element are

$$K = \frac{EA}{L^3} \begin{bmatrix} 12 & 6\,l_e & -12 & 6\,l_e \\ 6\,l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ 12 & -6l_e & 12 & -6l_e \\ -6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}$$

For the beam element, [5] we use the hermite shape function we have, v = hq on integrating, we get

mass matrix:

$$M = \frac{\rho A_e\, l_e}{420} \begin{bmatrix} 156 & 22\,l_e & 54 & -13 \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 156 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix}$$

where E is youngs modulus,

A is the cross sectional area,

l is element length.

ρ is density of the beam

combined eigen values and eigen vectors of undamped system are obtained using MATLAB software.

AV= λv

The statement,

[V, D] = eig (A, B) (6) ⟶

From the eigen value, we found the natural frequency values using the equation

$$f_n = \frac{w_n}{2\pi} \quad (7) \longrightarrow$$

III. Regression Method

The relationship between two or more dependent variables has been referred to as statistical determination of a correlation analysis, [6] whereas the determination of the relationship between dependent and independent variables has come to be known as a regression analysis.

3.1 Linear Regression:

The most straightforward methods for fitting a model to experimental data are those of linear regression. Linear regression involves specification of a linear relationship between the dependent variable(s) and certain properties of the system under investigation. Surprisingly though, linear regression deals with some curves (i.e., nonstraight lines) as well as straight lines, with regression of straight lines being in the category of "ordinary linear regression" and curves in the category of "multiple linear regressions" or "polynomial regressions."

3.2 Ordinary Linear Regression:

The simplest general model for a straight line includes a parameter that allows for inexact fits: an "error parameter" which we will denote as ε . Thus we have the formula:

$$Y = \alpha + \beta X + \varepsilon'$$

The parameter, α, is a *constant*, often called the "intercept" while b is referred to as a *regression coefficient* that corresponds to the "slope" of the line. The additional parameter, ε, accounts for the type of error that is due to random variation caused by experimental imprecision or simple fluctuations in the state of the system from one time point to another.

3.3 Multiple Linear Regressions:

The basic idea of the finite element method is piecewise approximation that is the solution of a complicated problem is obtained by dividing the region of interest into small regions (finite element) and approximating the solution over each sub region by a simple function. Thus a necessary and important step is that of choosing a simple function for the solution in each element. The functions used to represent the behavior of the solution within an element are called interpolation functions or approximating functions or interpolation models. Polynomial types of functions have been most widely used in the literature due to the following reasons.

(i) It is easier to formulate and computerize the finite element equations with polynomial functions. Specially it is easier to perform differentiation or integration with polynomials.

(ii) It is possible to improve the accuracy of the results by increasing the order of the polynomial. Theoretically a polynomial of infinite order corresponds to the exact solution. But in practice we use polynomials of finite order only as an approximation.

The interpolation or shape functions are expressed in terms of natural coordinates. The representation of geometry in terms of shape functions can be considered as a mapping procedure to calculate the natural frequency values for the variations of the physical properties.

Polynomial form of the shape functions for 1-D,2-D and 3-D elements are as follows:

$$\Phi(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \cdots + \alpha_m x^n \longrightarrow (8)$$

$$\Phi(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 y^2 + \alpha_6 xy \ldots + \alpha_m y^n \longrightarrow (9)$$

$$\Phi(x,y,z) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z + \alpha_5 x^2 + \alpha_6 y^2 + \alpha_7 z^2 + \alpha_8 xy + \alpha_9 yz + \alpha_{10} xz \ldots + \alpha_m z^n \longrightarrow (10)$$

3.4 Convergence Requirements:

Since the finite element method is a numerical technique, it obtains a sequence of approximate solutions as the element size is reduced successively. The sequence will converge to the exact solution if the polynomial function satisfies the following convergence requirements.

(i) The field variable must be continuous within the elements. This requirement is easily satisfied by choosing continuous functions as regression models. Since polynomials are inherently type of regression models as already discussed , satisfy the requriment.

(ii) All uniform states of the field variable 'Φ' and its partial derivatitives upto the highest order appearing in the function (Φ) must have representation in the polynomial when, in the limit, the sizes are increased (or) decreased successively.

(iii) The field variable 'Φ' and its partial derivatives up to one order less than the highest order derivative appearing in the field variable in the function (Φ) must be continuous at element boundaries or interfaces.

3.5 Non linear Regression:

A general model that encompasses all their behaviors cannot be defined in the sense used for linear models, so we can use an explicit nonlinear function for illustrative purposes.

In this case, we will use the Hill equation:

$$Y = \frac{\alpha[A]^s}{[A]^s + K^s} \longrightarrow (11)$$

Which contains one independent variable [A], and 3 parameters, α ,K, and S. Differentiating Y with respect to each model parameter yields the following:

$$\left.\begin{aligned} \frac{\partial y}{\partial \alpha} &= \frac{[A]^s}{[A]^s + K^s} \\ \frac{\partial y}{\partial K} &= \frac{-\alpha s (K[A])^s}{K([A]^s + K^s)^2} \\ \frac{\partial y}{\partial K} &= \frac{-\alpha s (K[A])^s}{K([A]^s + K^s)^2} \end{aligned}\right\} \longrightarrow (12)$$

All derivatives involve at least two of the parameters, so the model is nonlinear. However, it can be seen that the partial derivative in equation $\frac{\partial y}{\partial \alpha} = \frac{\alpha[A]^s}{[A]^s + K^s}$ does not contain the parameter, α.

However the model is linear because the first derivatives do not include the parameters. As a consequence, taking the second (or higher) order derivative of a linear function with respect to its parameters will always yield a value of zero. Thus, if the independent variables and all but one parameter are held constant, the relationship between the dependent variable and the remaining parameter will always be linear. It is important to note that linear regression does not actually test whether the data sampled from the population follow a linear relationship. It assumes linearity and attempts to find the best-fit straight line relationship based on the data sample. The dashed line shown in fig.(1) is the deterministic component, whereas the points represent the effect of random error.

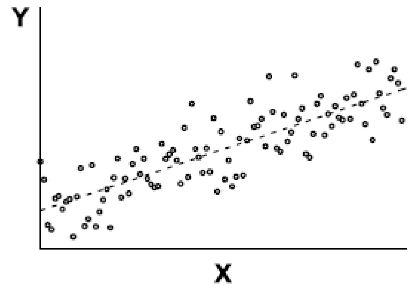

figure 1: a linear model that incorporates a stochastic (random error) component.

3.6 Assumptions of Standard Regression Analyses:

The subjects are randomly selected from a larger population. The same caveats apply here as with correlation analyses.

1. The observations are independent.
2. X and Y are not interchangeable. Regression models used in the vast majority of cases attempt to predict the dependent variable, Y, from the independent variable, X and assume that the error in X is negligible. In special cases where this is not the case, extensions of the standard regression techniques have been developed to account for non negligible error in X.
3. The relationship between X and Y is of the correct form, i.e., the expectation function (linear or nonlinear model) is appropriate to the data being fitted.
4. The variability of values around the line is Gaussian.
5. The values of Y have constant variance. Assumptions 5 and 6 are often violated (most particularly when the data has variance where the standard deviation increases with the mean) and have to be specifically accounted forin modifications of the standard regression procedures.
6. There are enough datapoints to provide a good sampling of the random error associated with the experimental observations. In general, the minimum number of independent points can be no less than the number of parameters being estimated, and should ideally be significantly higher.

IV. Numerical Examples

In finite element method **Discretization**, dividing the body into equivalent system of finite elements with associated nodes. Small elements are generally desirable where the results are changing rapidly such as where the changes in geometry occur. The element must be made small enough to view and give usable results and to be large enough to reduce computational efforts. Large elements can be used where the results are relatively constant. The discretized body or mesh is often created with mesh generation program or preprocessor programs available to the user.

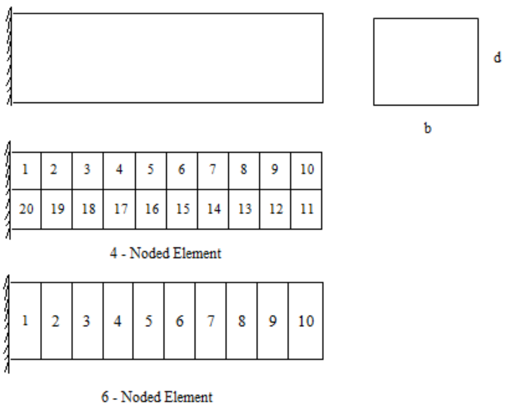

Figure 2: descretized element

The polynomial regression equation for a quadratic element is,

$f_n = \alpha_1 + \alpha_2 B + \alpha_3 H + \alpha_4 B^2 + \alpha_5 H^2 + \alpha_6 BH$

The values of young's modulus(E), density(ρ), length(l) breadth(b), depth(d) for the both case studies are as follows:

Young's modulus(E)	0.207×10^{12} N/m^2
Density(ρ)	7806 Kg/m^3
Length(l)	0.45m
Breadth(b)	0.02m
Depth(d)	0.003m

1.1 Case study 1:

The cantilever beam of 0.45m length, shown in fig.(3) is divided into 10 elements equally. Element stiffness matrix and mass matrix for each element are extracted and natural frequencies of cantilever beam are calculated by considering the following situations:

i. Increasing the depth(d) of the beam alone by 5%
ii. Increasing the breadth(b) and depth(d) of the beam by 5%
iii. Decreasing the depth(d) of the beam alone by 5%
iv. Decreasing the breadth(b) and depth(d) of the beam by 5%

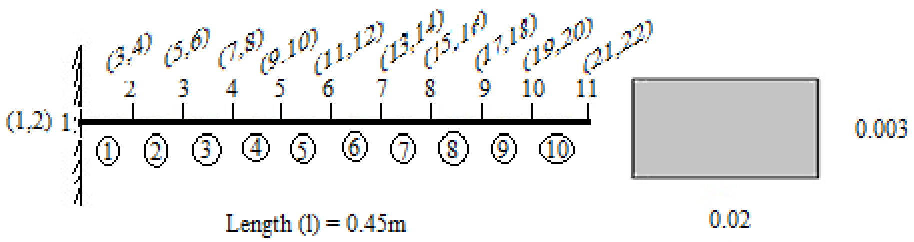

Figure 3: cantilever beam

Reanalysis of the beam is done by using polynomial regression method and the percentage errors are listed in the tabular column.

First natural frequencies of cantilever beam from polynomial regression for Increasing the depth(d) alone by 5% are as follows:

$f_n = \alpha_1 + \alpha_2 B + \alpha_3 H + \alpha_4 B^2 + \alpha_5 H^2 + \alpha_6 BH$

Fitting target of lowest sum of squared absolute error $= 3.6923449893443150E-05$

$\alpha_1 = 3.6862173276153008E-02$ $\alpha_2 = 7.3719517752124375E-04$

$\alpha_3 = 4.0852990583098785E+03$ $\alpha_4 = 1.4744871051242114E-05$

$\alpha_5 = 2.7402227402200751E+03$ $\alpha_6 = 8.1705981166207820E+01$

Table 1: Increasing the depth(d) of the beam alone by 5%

Breadth(b)	Depth(d)	f_n (FEM)	f_n (Regression)	% Error
0.02	0.003	12.322	12.32234	0.00274681
0.02	0.00315	12.9386	12.93791	-0.00536153
0.02	0.0033	13.5547	13.5536	-0.00813418
0.02	0.00345	14.17	14.16941	-0.0041504
0.02	0.0036	14.78	14.78535	0.0361952
0.02	0.00375	15.403	15.40141	-0.01031797
0.02	0.0039	16.019	16.0176	-0.00877018
0.02	0.00405	16.635	16.6339	-0.00659575
0.02	0.0042	17.25	17.25033	0.00193507
0.02	0.00435	17.867	17.86689	-0.00062623
0.02	0.0045	18.483	18.48357	0.00306084

First natural frequencies of cantilever beam from polynomial regression for Increasing the breadth(b) and depth(d) by 5% are as follows:

Fitting target of lowest sum of squared absolute error = $3.5049883449909422E-05$

$\alpha_1 = 3.3657342657326061E-02$ $\alpha_2 = 5.9975538723705108E+02$

$\alpha_3 = 8.9963308085557543E+01$ $\alpha_4 = 5.6964518325322445E+01$

$\alpha_5 = 1.2817016623197541E+00$ $\alpha_6 = 8.5446777487983585E+00$

Table 2: Increasing the breadth(b) and depth(d) of the beam by 5%

Breadth(b)	Depth(d)	f_n (FEM)	f_n (Regression)	% Error
0.02	0.003	12.322	12.32197	-0.00028
0.021	0.00315	12.9278	12.9376	0.075838
0.022	0.0033	13.5047	13.55336	0.360318
0.023	0.00345	14.12	14.16923	0.34867
0.024	0.0036	14.58	14.78522	1.407551
0.025	0.00375	15.385	15.40133	0.106119
0.026	0.0039	15.989	16.01755	0.178549
0.027	0.00405	16.434	16.63389	1.2163
0.028	0.0042	17.18	17.25034	0.40944
0.029	0.00435	17.854	17.86691	0.072327
0.03	0.0045	18.264	18.4836	1.202373

First natural frequencies of cantilever beam from polynomial regression for Decreasing the depth(d) alone by 5% are as follows:

Fitting target of lowest sum of squared absolute error = $1.0898007711580207E-04$

$\alpha_1 = -5.1051580269430227E-02$ $\alpha_2 = -1.0208111928022845E-03$

$\alpha_3 = 4.1617777597585637E+03$ $\alpha_4 = -2.0420625725492414E-05$

$\alpha_5 = -1.2279202279201156E+04$ $\alpha_6 = 8.3235555195174001E+01$

Table 1: Decreasing the depth(d) of the beam alone by 5%

Breadth(b)	Depth(d)	f_n (FEM)	f_n (Regression)	% Error
0.02	0.003	12.322	12.32874	0.05472
0.02	0.00285	11.72095	11.715	-0.05075
0.02	0.0027	11.10406	11.10071	-0.03019
0.02	0.00255	10.48716	10.48586	-0.01239
0.02	0.0024	9.87027	9.870462	0.001942
0.02	0.00225	9.25338	9.25451	0.012213
0.02	0.0021	8.636	8.638006	0.023227
0.02	0.00195	8.0195	8.020949	0.01807
0.02	0.0018	7.402	7.40334	0.018101
0.02	0.00165	6.785	6.785178	0.002623
0.02	0.0015	6.1689	6.166463	-0.0395

First natural frequencies of cantilever beam from polynomial regression for Decreasing the breadth(b) and depth(d) by 5% are as follows:

Fitting target of lowest sum of squared absolute error = $1.0898007701638626E-04$

$\alpha_1 = -5.1072004661949374E-02$ $\alpha_2 = 6.1077395660573734E+02$

$\alpha_3 = 9.1616093490849153E+01$ $\alpha_4 = -2.7006878138023399E+02$

$\alpha_5 = -6.0765475810552818E+00$ $\alpha_6 = -4.0510317207035214E+01$

Table 4: Decreasing the breadth(b) and depth(d) of the beam by 5%

Breadth(b)	Depth(d)	f_n (FEM)	f_n (Regression)	% Error
0.02	0.003	12.322	12.08138	-1.95277
0.019	0.00285	11.71924	11.48001	-2.04138
0.018	0.0027	11.09845	10.87808	-1.98559
0.017	0.00255	10.42961	10.2756	-1.47664
0.016	0.0024	9.8594	9.672571	-1.89493
0.015	0.00225	9.21894	9.068988	-1.62657
0.014	0.0021	8.614	8.464851	-1.73147
0.013	0.00195	8.0098	7.860163	-1.86818
0.012	0.0018	7.397	7.254922	-1.92075
0.011	0.00165	6.693	6.649128	-0.65549
0.01	0.0015	6.1562	6.042782	-1.84234

1.2 Case study 2:

The **T**-structure having dimensions as shown in fig.(4), is divided into 5 elements equally. Element stiffness matrix and mass matrix for each element are extracted and natural frequencies of structure are calculated by considering the situations which have been taken in 4.1:

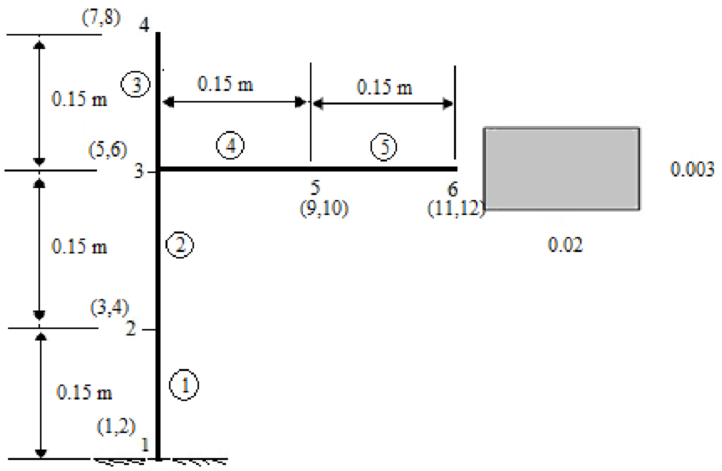

Figure 4: T-structure

Reanalysis of the beam is done by using polynomial regression method and the percentage errors are listed in the tabular column.

First natural frequencies of cantilever beam from polynomial regression for Increasing the depth(d) alone by 5% are as follows:

$$f_n = \alpha_1 + \alpha_2 B + \alpha_3 H + \alpha_4 B^2 + \alpha_5 H^2 + \alpha_6 BH$$

Fitting target of lowest sum of squared absolute error $= 7.4951981373616095E-07$

$\alpha_1 = -1.6182386629584215E-04$ $\quad$ $\alpha_2 = -3.2093375921249390E-06$

$\alpha_3 = 1.8476394671346783E+05$ $\quad$ $\alpha_4 = -6.4730613758001709E-08$

$\alpha_5 = -6.4750064845755716E+01$ $\quad$ $\alpha_6 = 3.6952789342693527E+03$

Table 5: Increasing the depth(d) of the beam alone by 5%

Breadth(b)	Depth(d)	f_n (FEM)	f_n (Regression)	% Error
0.02	0.003	554.513	554.5128	-2.48438E-05
0.02	0.00315	582.238	582.2384	7.3913E-05
0.02	0.0033	609.9642	609.964	-2.53215E-05
0.02	0.00345	637.6898	637.6897	-2.22942E-05
0.02	0.0036	665.4155	665.4153	-3.49852E-05
0.02	0.00375	693.141	693.1409	-1.82272E-05
0.02	0.0039	720.866	720.8665	6.61985E-05
0.02	0.00405	748.592	748.5921	1.03971E-05
0.02	0.0042	776.318	776.3177	-4.17937E-05
0.02	0.00435	804.043	804.0433	3.36238E-05
0.02	0.0045	831.769	831.7689	-1.65626E-05

First natural frequencies of cantilever beam from polynomial regression for Increasing the breadth(b) and depth(d) by 5% are as follows:

Fitting target of lowest sum of squared absolute error = $7.4951981339546874E-07$

$\alpha_1 = -1.6188811317291245E-04$ $\alpha_2 = 2.7115577353372031E+04$

$\alpha_3 = 4.0673366030058064E+03$ $\alpha_4 = 4.0673366030058064E+00$

$\alpha_5 = -3.2042541568134908E-02$ $\alpha_6 = -2.1361694378748552E-04$

Table 6: Increasing the breadth(b) and depth(d) of the beam by 5%

Breadth(b)	Depth(d)	f_n (FEM)	f_n (Regression)	% Error
0.02	0.003	554.513	554.5128	-2.48438E-05
0.021	0.00315	582.238	582.2384	7.3913E-05
0.022	0.0033	609.9642	609.964	-2.53215E-05
0.023	0.00345	637.6898	637.6897	-2.22942E-05
0.024	0.0036	665.4155	665.4153	-3.49852E-05
0.025	0.00375	693.141	693.1409	-1.82272E-05
0.026	0.0039	720.866	720.8665	6.61985E-05
0.027	0.00405	748.592	748.5921	1.03971E-05
0.028	0.0042	776.318	776.3177	-4.17937E-05
0.029	0.00435	804.043	804.0433	3.36238E-05
0.03	0.0045	831.769	831.7689	-1.65626E-05

First natural frequencies of cantilever beam from polynomial regression for Decreasing the depth(d) alone by 5% are as follows:

Fitting target of lowest sum of squared absolute error = $4.8406526812429600E-07$

$\alpha_1 = 4.5098480533081574E-04$ $\alpha_2 = 92209871374071.032E-06$

$\alpha_3 = 1.8476295891899648E+05$ $\alpha_4 = 1.8039435190075892E-07$

$\alpha_5 = 2.0461020463086680E+02$ $\alpha_6 = 3.6952591783799307E+03$

Table 7: Decreasing the depth(d) of the beam alone by 5%

Breadth(b)	Depth(d)	f_n (FEM)	f_n (Regression)	% Error
0.02	0.003	554.5128	554.5128	1.42609E-05
0.02	0.00285	526.787	526.7871	2.46623E-05
0.02	0.0027	499.0616	499.0614	-3.4071E-05
0.02	0.00255	471.336	471.3357	-5.53281E-05
0.02	0.0024	443.61	443.6101	1.30024E-05
0.02	0.00225	415.884	415.8844	9.26578E-05
0.02	0.0021	388.159	388.1587	-7.1562E-05
0.02	0.00195	360.433	360.4331	1.89521E-05
0.02	0.0018	332.7077	332.7074	-8.30757E-05
0.02	0.00165	304.982	304.9818	-6.94796E-05
0.02	0.0015	277.256	277.2562	5.83598E-05

First natural frequencies of cantilever beam from polynomial regression for Decreasing the breadth(b) and depth(d) by 5% are as follows:

Fitting target of lowest sum of squared absolute error = $4.8406526798552702E-07$

$\alpha_1 = 4.5116550270616755E-04$ $\alpha_2 = 2.7115432386684053E+04$

$\alpha_3 = 4.0673148580025731E+03$ $\alpha_4 = 4.5001969533233819E+00$

$\alpha_5 = 1.0125443144973012E-01$ $\alpha_6 = 6.7502954299811790E-01$

Table 8: Decreasing the breadth(b) and depth(d) of the beam by 5%

Breadth(b)	Depth(d)	f_n (FEM)	f_n (Regression)	% Error
0.02	0.003	554.5128	554.5129	2.2536E-05
0.019	0.00285	526.787	526.7872	3.3373E-05
0.018	0.0027	499.0616	499.0615	-2.4877E-05
0.017	0.00255	471.336	471.3358	-4.5593E-05
0.016	0.0024	443.61	443.6101	2.3346E-05

0.015	0.00225	415.884	415.8844	0.00010369
0.014	0.0021	388.159	388.1588	-5.974E-05
0.013	0.00195	360.433	360.4331	3.1683E-05
0.012	0.0018	332.7077	332.7075	-6.9284E-05
0.011	0.00165	304.982	304.9818	-5.4434E-05
0.01	0.0015	277.256	277.2562	7.491E-05

V. Conclusion

From this work the following results are drawn. Natural frequencies are obtained for dynamic analysis of cantilever beam and T-structure from FEM using MATLAB and Polynomial regression method by considering the situations mentioned in 4.1. The maximum and minimum errors are obtained when the results of Regression method are compared with FEM.

Situations – Parameters increased (or) decreased by 5%	Cantilever Beam		T-Structure	
	Maximum	Minimum	Maximum	Minimum
1. Increasing i. Depth ii. Breadth(b) and Depth(d)	0.0361952 1.407551	-0.00062623 -0.00028	7.3913E-05 7.3913E-05	-4.17937E-05 -4.17937E-05
2. Decreasing i. Depth ii. Breadth(b) and Depth(d)	0.023227 -0.65549	-0.05075 -2.04138	9.26578E-05 7.491E-05	-8.30757E-05 -6.9284E-05

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