IOSR Journal of Polymer and Textile Engineering (IOSR-JPTE)
e-ISSN: 2348-019X, p-ISSN: 2348-0181, Volume 6, Issue 3 (May. - June. 2019), PP 07-13
www.iosrjournals.org

Neutrosophic Feebly Connectedness and Compactness

Dr.K.Bageerathi[1], P. Jeya Puvaneswari[2]
[1]*Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur - 628216.*
[2]*Department of Mathematics, Vivekananda College, Agasteeswaram – 629701*
Corresponding Author: Dr.K.Bageerathi

Abstract: *The aim of this paper is to construct the concepts related to connectedness in neutrosophic topological spaces. Here we introduce the concepts of neutrosophic feebly connected, neutrosophic feebly disconnected, neutrosophic feebly C_i - disconnected* $(i = 1,2,3,4)$, *neutrosophic feebly* $C_i -$ *connected* $(i = 1,2,3,4)$ *and neutrosophic feebly compact in neutrosophic topological spaces and obtain several preservation properties and some characterizations concerning connectedness in these spaces.*

AMS Subject Classification: 03E72

Keywords and Phrases: *Neutrosophic topology, Neutrosophic feebly connected, Neutrosophic feebly disconnected, Neutrosophic feebly C_i-disconnected$(i = 1, 2, 3, 4)$, Neutrosophic feebly C_i − connected $(i = 1, 2, 3, 4)$ and Neutrosophic feebly compact.*

Date of Submission: 08-05-2019 Date of acceptance: 24-05-2019

I. Introduction

Ever since the introduction of fuzzy sets by L.A.Zadeh [17], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by C.L.Chang [3]. Atanassov[2] introduced the notion of intuitionistic fuzzy sets, Coker [4] introduced the intuitionistic fuzzy topological spaces. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Turnali and Coker[15]. The neutrosophic set was introduced by Smarandache [15] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. In 2012, Salama, Alblowi [1, 13-15], introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. In 2014, Salama,Smarandache and Valeri [16] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions.

Iswarya et al. [9] defined the concept of neutrosophic semi open sets in neutrosophic topological spaces. Jeyapuvaneswari et al. [10-12] defined neutrosophic feebly open sets, neutrosophic feebly closed setsand neutrosophic feebly continuous functions in neutrosophic topological spaces.

In this paper, the concepts of neutrosophic feebly connected, neutrosophic feebly disconnected, neutrosophic feebly C_i-connected$(i = 1,2,3,4)$, neutrosophic feebly $C_i -$ disconnected $(i = 1,2,3,4)$ and neutrosophic feebly compact spaces are discussed in neutrosophic topological spaces and studied several preservation properties and some characterizations concerning connectedness in these spaces.

This paper is organized as follows. Section II gives the basic definitions of neutrosophic feebly open sets, neutrosophic feebly closed sets and their properties which are used in the later sections. The Section III deals with the concept of neutrosophic feebly connected, neutrosophic feebly disconnected spaces, neutrosophic feebly C_i-connected$(i = 1,2,3,4)$, and neutrosophic feebly $C_i -$ disconnected $(i = 1,2,3,4)$ spaces.Section IV explains neutrosophic feebly compact in neutrosophic topological spaces and their properties.

II. Preliminaries

First we shall present the fundamental definitions. The following one is obviously inspired bySmarandache [5-7] and Salama[13-15]

Definition 2.1. [15] Let Xbe a non-empty fixed set. A neutrosophic set A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Neutrosophic sets 0_N and 1_N in X as follows:
0_N may be defined as:
(0_1) $0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X\}$
(0_2) $0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X\}$
(0_3) $0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X\}$
(0_4) $0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X\}$
1_N may be defined as:
(1_1) $1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X\}$
(1_2) $1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X\}$
(1_3) $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X\}$
(1_4) $1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X\}$

Definition 2.2. [15] Let $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ be a neutrosophic set in X. Then the complement of the set A[A^C for short] may be defined as three kinds of complements:
(C_1) $A^C = \{ \langle x, 1-\mu_A(x), 1-\sigma_A(x), 1-\gamma_A(x) \rangle : x \in X\}$
(C_2) $A^C = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X\}$
(C_3) $A^C = \{ \langle x, \gamma_A(x), 1-\sigma_A(x), \mu_A(x) \rangle : x \in X\}$
One can define several relations and operations between neutrosophic setsfollows:

Definition 2.3. [15] Let $x$be a non-empty set, and neutrosophic sets $A$and Bin the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$ and $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X\}$. Then we may consider two possible definitions for subsets ($A \leq B$).
$A \leq B$ may be defined as:
(1) $A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) and\ \gamma_A(x) \geq \gamma_B(x), \forall x \in X$
(2) $A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x) and\ \gamma_A(x) \geq \gamma_B(x), \forall x \in X$

Definition 2.4. [15] Let Xbe a non-empty set, and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$, $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ are neutrosophic subsets. Then
(1) $A \wedge B$ may be defined as:
(I_1) $A \wedge B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\gamma_A(x) \vee \gamma_B(x) \rangle$
(I_2) $A \wedge B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\gamma_A(x) \vee \gamma_B(x) \rangle$
(2) $A \vee B$ may be defined as:
(U_1) $A \vee B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\gamma_A(x) \wedge \gamma_B(x) \rangle$
(U_2) $A \vee B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\gamma_A(x) \wedge \gamma_B(x) \rangle$
We can easily generalize the operations of intersection and union in Definition 2.4 to arbitrary family of neutrosophic setsas follows:

Definition 2.5. [15] Let $\{A_j : j \in J\}$ be an arbitrary family of neutrosophic sets in X. Then
(1) $\wedge A_j$may be defined as:
(i) $\wedge A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \wedge_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \gamma_{A_j}(x) \rangle$
(ii) $\wedge A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \vee_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \gamma_{A_j}(x) \rangle$
(2) $\vee A_j$may be defined as:
(i) $\vee A_j = \langle x, \vee_{j \in J} \mu_{A_j}, \vee_{j \in J} \sigma_{A_j}, \wedge_{j \in J} \gamma_{A_j} \rangle$
(ii) $\vee A_j = \langle x, \vee_{j \in J} \mu_{A_j}, \vee_{j \in J} \sigma_{A_j}, \wedge_{j \in J} \gamma_{A_j} \rangle$

Definition 2.6.[9]Let $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$, $B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y) \rangle$ be neutrosophic subsets of X and Y respectively. Then $A \times B$ is a neutrosophic set of $X \times Y$ is defined by

$$(P_1)(A \times B)(x, y) = \langle (x, y), min\ (\mu_A(x), \mu_B(y)), min\ (\sigma_A(x), \sigma_B(y)), max\ (\gamma_A(x), \gamma_B(y)) \rangle$$
$$(P_2)(A \times B)(x, y) = \langle (x, y), min\ (\mu_A(x), \mu_B(y)), max\ (\sigma_A(x), \sigma_B(y)), max\ (\gamma_A(x), \gamma_B(y)) \rangle$$

Notice that

$$(CP_1)(A \times B)^C(x, y) = \langle (x, y), max\ (\mu_A(x), \mu_B(y)), max\ (\sigma_A(x), \sigma_B(y)), min\ (\gamma_A(x), \gamma_B(y)) \rangle$$
$$(CP_2)(A \times B)^C(x, y) = \langle (x, y), max\ (\mu_A(x), \mu_B(y)), min\ (\sigma_A(x), \sigma_B(y)), min\ (\gamma_A(x), \gamma_B(y)) \rangle$$

Definition 2.7. [15] A neutrosophic topology [NT for short] is a non-empty set $X$is a family τ of neutrosophic subsets in Xsatisfying the following axioms:
(NT_1) $0_N, 1_N \in \tau$,
(NT_2) $G_1 \wedge G_2 \in \tau$for any $G_1, G_2 \in \tau$,
(NT_3) $\vee G_i \in \tau$for every $\{G_i : i \in J\} \leq \tau$
In this case the pair (X, τ) is called a neutrosophic topological space. The elements of τ are called neutrosophic open sets.

Definition 2.8. [15] The complement of A [A^C for short] of neutrosophic open setsis called a neutrosophic closed set in X.
Definition 2.9. [10] A neutrosophic subset $A=\langle x, \mu_A, \sigma_A, \gamma_A\rangle$ in a neutrosophic topological space (X, τ) is said to be a neutrosophic neighborhood of a neutrosophic point $x_{(r,t,s)} \in X$, if there exists a neutrosophic open set $B = \langle x, \mu_B, \sigma_B, \gamma_B\rangle$with $x_{r,t,s} \leq B \leq A$.
Now, we define neutrosophic closure and neutrosophic interior operations in neutrosophic topological spaces:
Definition 2.10[16]Let X and Y be two nonempty neutrosophic sets and f : X → Y be a function.
(i) If B = {(y, $\mu_B(y)$, $\sigma_B(y)$, $\gamma_B(y)$): y ∈Y} is a Neutrosophic set in Y , then the pre image of B under f is denoted and defined by $f^{1}(B)$ = { (x, $f^{-1}(\mu_B)(x)$, $f^{-1}(\sigma_B)(x)$, $f^{-1}(\gamma_B)(x)$) : x ∈ X }.
(ii) If A = { <x, $\alpha_A(x)$, $\delta_A(x)$, $\lambda_A(x)$) : x ∈ X } is a NS in X , then the image of A under f is denoted and definedby f (A) = {(y, f (α_A)(y), f (δ_A)(y), f _(λ_A)(y)): y ∈ Y } where f_(λ_A) = C (f (C (A))).
In (i), (ii), since μ_B, σ_B, γ_B, α_A, δ_A, $\lambda_A$are neutrosophic sets, we explain that $f^{-1}(\mu_B)(x) = \mu_B(f(x))$,
andf (α_A)(y) = $= \begin{cases} \sup\{\alpha_A(x) : x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \varphi \\ 0 & \text{otherwise} \end{cases}$
Lemma 2.11. [16]Let $f: X \to Y$ be a function. If A is a neutrosophic subset of X and μ is a neutrosophic subset of Y. Then
(i) $f(f^{-1}(A)) \leq A$
(ii) $f(f^{-1}(A)) = A \Leftrightarrow f$ is surjective.
$(iii) f^{-1}(f(A)) \geq A$
(iv) $f^{-1}(f(A)) = A$ whenever f is injective.
Definition 2.12.[10] A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly open if there is a neutrosophic open set U in X such that $U \leq A \leq NScl(U)$.
Lemma 2.13. [10] **(i)** Every neutrosophic open set is a neutrosophic feebly open set.
Definition 2.14. [10]A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly closed if there is a neutrosophic closed set U in X such that $NSint(U) \leq A \leq U$.
Lemma 2.15. [10]A neutrosophic subset A of a neutrosophic topological space (X, τ) is
(i) neutrosophic feebly closed if and only if $Ncl\left(Nint(Ncl(A))\right) \leq A$.
(ii) A neutrosophic subset A is neutrosophic feebly closed iff if $NSint(Ncl(A)) \leq A$.
(iii) A neutrosophic subset A is a neutrosophic feebly closed set if and only if A^c is neutrosophic feebly open.
(iv) Every neutrosophic closed set is a neutrosophic feebly closed set.
Definition 2.16. [10]Let (X, τ) be neutrosophic topological space and $A=\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ be a neutrosophic set in X. Then neutrosophic feebly interior of A is defined by $NFint(A) = \vee\{G: G$is a neutrosophic feebly open set in X and $G \leq A\}$.
Lemma 2.17.[10]Let (X, τ) be neutrosophic topological space. Then for any neutrosophic feebly subsets A and B of a neutrosophic topological space$X$, we have
(i) $NFint(A) \leq A$
(ii) $A$is neutrosophic feebly open set in $X \Leftrightarrow NFint(A) = A$
Definition 2.18.[10]Let (X, τ) be neutrosophic topological space and$A= \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ be a neutrosophic set in X. Then the neutrosophic feebly closure is defined by $NFcl(A)= \wedge\{K: K$is a neutrosophic feebly closed set in X and $A \leq K\}$.
Lemma 2.19. [10]Let (X, τ) be a neutrosophic topological space. Then for any neutrosophic subset $A$of X,
(i) $(NFint(A))^c = NFcl(A^c)$
(ii) $(NFcl(A))^c = NFint(A^c)$.
Lemma 2.20.[10]Let (X, τ) be a neutrosophic topological space. Then for any neutrosophic subsets A and B of a neutrosophic topological space X,
(i) $A \leq NFcl(A)$
(ii) $A$is a neutrosophic feebly closed set in $X \Leftrightarrow NFcl(A) = A$
(iii) If $A \leq B$then $NFcl(A) \leq NFcl(B)$.
Definition 2.21. [11] Let (X, τ) and (Y, σ) be neutrosophic topological spaces. Then a map $f: (X, \tau) \to (Y, \sigma)$ is called neutrosophic continuous (in short N-continuous) function if the inverse image of every neutrosophic open set in (Y, σ) is neutrosophic open set in (X, τ).

Definition 2.22.[12]Let $(X,\tau)$and (Y,σ) be two neutrosophic topological spaces. A function $f: X \to Y$ is called neutrosophic feebly irresolute if the inverse image of every neutrosophic feebly open set in Y is neutrosophic feebly open in X.

III. Neutrosophic Feebly Connected Spaces

In this section, we study the notion of neutrosophic feebly connected, neutrosophic feebly disconnected in neutrosophic topological spaces. Also, we introduced neutrosophic feebly C_i-connected $(i = 1,2,3,4)$and neutrosophic feebly C_i-disconnected $(i = 1,2,3,4)$ *and we give some properties and theorems of such concepts.*

Definition 3.1.A neutrosophic topological space (X,τ) is neutrosophic feebly disconnected if there exists neutrosophic feebly open sets A, Bin $X, A \neq 0_N$, B $\neq 0_N$ such that $A \vee B = 1_N$ and $A \wedge B = 0_N$.

That is (i) $< x,\ \mu_A \vee \mu_B,\ \sigma_A \wedge \sigma_B,\ \gamma_A \wedge \gamma_B > = 1_N$.

(ii)$< x, \mu_A \vee \mu_B,\ \sigma_A \vee \sigma_B,\ \gamma_A \wedge \gamma_B > = 1_N$

(iii) $< x, \mu_A \wedge \mu_B,\ \sigma_A \wedge \sigma_B,\ \gamma_A \vee \gamma_B > = 0_N$

(iv)$< x, \mu_A \wedge \mu_B,\ \sigma_A \vee \sigma_B,\ \gamma_A \vee \gamma_B > = 0_N$

The following example shows that X is neutrosophic feebly disconnected space.

Example 3.2. Let $X = \{a,b,c\}$, τ = $\{0_N, 1_N, A, B, C, D\}$ where

$$A = \{< x, (0,0,0), (0,0,1), (1,0,1) >\}$$
$$B = \{< x, (1,0,0), (1,1,0), (0,0,0) >\}$$
$$C = \{< x, (1,0,0), (1,1,0), (1,0,0) >\}$$
$$D = \{< x, (0,0,0), (0,0,1), (0,0,1) >\}$$

A and B are neutrosophic feebly open sets in $X, A \neq 0_N, B \neq 0_N$, and $A \vee B = C = 1_{\mathrm{N}}, A \wedge B = D = 0_{\mathrm{N}}$.
Hence X is neutrosophic feebly disconnected space.

If X is not neutrosophic feebly disconnected then it is said to be neutrosophic feebly connected.

Example 3.3. Let $X = \{a,b\}$,τ =$\{0_N$,$1_N, G_1\} where G_1 = \{\ x, (0.2, 0.1), (0.7, 0.5), (0.7, 0.5) >\}$. Then (X,τ) is a neutrosophic topological space. Let $G_2 = \{< x, (0.3, 0.2), (0.6,0.4), (0.6,0.4) >\}$. Then $G_1$and G_2 are neutrosophic feebly open sets in X, $G_1 \neq 0_N$, $G_2 \neq 0_N$, and $G_1 \vee G_2 = G_2 \neq 1_{\mathrm{N}}, G_1 \wedge G_2 = G_1 \neq 0_{\mathrm{N}}$. Hence X is neutrosophic feebly connected.

Example 3.4. Let $= \{a,b\}$, τ = $\{0_N, 1_N, G_1\}$ where$G_1 = \{\ x, (0.2, 0.1), (0.7, 0.5), (0.7, 0.5) >\}$. Then (X,τ) is a neutrosophic topological space. Let $G_2 = \{< x, (1,0), (0,1), (0,1) >\}$and $G_3 = \{< x, (0,1), (1,0), (1,0) >\}$. It can be found that $G_2$and G_3 are neutrosophic feebly open sets in X, $G_2 \neq 0_{\mathrm{N}}, G_3 \neq 0_{\mathrm{N}}$, and $G_2 \vee G_3 = 1_{\mathrm{N}},\ G_2 \wedge \mathrm{G}_3 = 0_{\mathrm{N}}$. Hence X is neutrosophic feebly disconnected.

Definition 3.5. Let N be a neutrosophic subset in neutrosophic topological space(X,τ)

(a) If there exists neutrosophic feebly open sets U and V in X satisfying the following properties, then N is called neutrosophic feebly C_i - disconnected $(i = 1,2,3,4)$:

.C_1: $N \leq U \vee V,\ U \wedge V \leq N^c, N \wedge U \neq 0_N, N \wedge V \neq 0_N$

.C_2: $N \leq U \vee V, N \wedge U \wedge V = 0_N, N \wedge U \neq 0_N, N \wedge V \neq 0_N$.

.C_3: $N \leq U \vee V, U \wedge V \leq N^c, U \nless N^c, V \nless N^c$

.C_4: $N \leq U \vee V, N \wedge U \wedge V = 0_N, U \nless N^c, V \nless N^c$

(b)$N$is said to be neutrosophic feebly C_i-connected $(i = 1,2,3,4)$ if N is not neutrosophic feebly C_i-disconnected $(i = 1,2,3,4)$.

Obviously, we can obtain the following implications between several types of neutrosophic feebly C_i-connected $(i = 1,2,3,4)$.

1. neutrosophic feebly C_1- connected⇒neutrosophic feebly C_2- connected
2. neutrosophic feebly C_1- connected⇒neutrosophic feebly C_3- connected
3. neutrosophic feebly C_3- connected⇒neutrosophic feebly C_4- connected
4. neutrosophic feebly C_2- connected⇒neutrosophic feebly C_4- connected

Example 3.6. Let $= \{a,b\}$, τ = $\{0_N, 1_N, G_1, G_2\}$ where

$$G_1 = \{< x, (0.4, 0.1), (0.6, 0.9), (0.6, 0.9) >\}$$
$$G_2 = \{< x, (0.5, 0.3), (0.5,0.7), (0.5, 0.7) >\}$$

Consider the neutrosophic subset $G_3 = \{< x, (0.3, 0.1), (0.7, 0.9), (0.7, 0.9) >\}$. Then$G_3$isneutrosophic feebly C_2-connected, neutrosophic feebly C_3-connected, neutrosophic feebly C_4- connectedbut neutrosophic feebly C_1- disconnected.

Example 3.7. Let $= \{a,b\}$, τ = $\{0_N, 1_N, G_1, G_2, G_1 \vee G_2, G_1 \wedge G_2\}$ where

$$G_1 = \{< x, (0.2, 0.8), (0.6, 0.2), (0.6, 0.2) >\}$$
$$G_2 = \{< x, (0.8, 0.6), (0.2, 0.2), (0.2, 0.2) >\}$$

Consider the neutrosophic subset $G_3 = \{< x, (0.1, 0.1), (0.9, 0.9), (0.9, 0.9) >\}$. Then$G_3$isneutrosophic feebly C_4-connected but neutrosophic feebly C_3-disconnected.

Definition 3.8. A neutrosophic topological space(X, τ) is neutrosophic feebly C_5-disconnected if there exists neutrosophic subset A in X which is both neutrosophic feebly open and neutrosophic feebly closed in X, such that $A \neq 0_N$, $A \neq 1_N$. If X is not neutrosophic feebly C_5-disconnected then it is said to be neutrosophic feebly C_5-connected.

Example 3.9. Let $= \{a, b\}$, τ = $\{0_N, 1_N, G_1\}$ where
$G_1 = \{< x, (0.2, 0.6), (0.5, 0.5), (0.2, 0.6) >\}$. Then $G_1$is neutrosophic feebly open and neutrosophic feebly closed set in X, $G_1 \neq 0_N$, $G_1 \neq 1_N$. Thus X is neutrosophic feebly C_5-connected.

Theorem 3.10. Neutrosophic feebly C_5-disconnectedness implies neutrosophic feebly connectedness.

Proof. Suppose that there exists non-empty neutrosophic feebly open sets A and B such that $A \vee B = 1_N$and $A \wedge B = 0_N$. Then $\mu_A \vee \mu_B = 1_N, \sigma_A \wedge \sigma_B = 0_N, \gamma_A \wedge \gamma_B = 0_Nand\mu_A \vee \mu_B = 0_N, \sigma_A \wedge \sigma_B = 1_N, \gamma_A \wedge \gamma_B = 1_N$. In other words, $B^c = A$. Hence A is neutrosophic feebly clopenwhich implies X is neutrosophic feebly connected.

But the converse may not be true as shown by the following example.

Example 3.11. Let $= \{a, b\}$, τ = $\{0_N, 1_N, G_1, G_2\}$ where$G_1 = \{< x, (0.2, 0.1), (0.7, 0.5), (0.6, 0.9) >\}$ and $G_2 = \{< x, (0.3, 0.1), (0.2, 0.6), (0.2, 0.3) >\}$. Then $G_1$and G_2 are neutrosophic feebly open sets in X. $G_1 \vee G_2 = \{< x, (0.6, 0.6), (0.2, 0.6), (0.2, 0.3) >\} \neq 1_N$. Then
$G_1 \wedge G_2 = \{< x, (0.2, 0.6), (0.6, 0.6), (0.6, 0.9) >\} \neq 0_N$. Hence X is neutrosophic feebly connected.Since neutrosophic subset G_1 is both neutrosophic feebly open and neutrosophic feebly closed in X, X is neutrosophic feebly C_5-disconnected.

Theorem 3.12. Let $f: (X, \tau) \to (Y, \sigma)$be a neutrosophic feebly irresolute surjection and Xbe neutrosophic feebly connected. Then $Y$is neutrosophic feebly connected.

Proof. Assume that $Y$is not neutrosophic feebly connected, then there exists nonempty neutrosophic feebly open sets U and V in Ysuch that $U \vee V = 1_N$and $U \wedge V = 0_N$. Since $f$is neutrosophic feebly irresolute mapping, $A = f^{-1}(U) \neq 0_N, B = f^{-1}(V) \neq 0_N$, which are neutrosophic feebly open sets in X and $f^{-1}(U) \vee f^{-1}(V) = f^{-1}(1_N) = 1_N$, which implies $A \vee B = 1_N$. Also $f^{-1}(U) \wedge f^{-1}(V) = f^{-1}(0_N) = 0_N$, which implies $A \wedge B = 0_N$. Thus X is neutrosophic feebly disconnected, which is a contradiction to our hypothesis. Hence Y is neutrosophic feebly connected.

Theorem 3.13. A neutrosophic topological space(X, τ) is neutrosophic feeblyC_5-connected if and only if there exists no nonempty neutrosophic feebly open sets U and V in X such that $U = V^c$.

Proof. Suppose that U and V areneutrosophic feebly open sets in X such that $U \neq 0_N, V \neq 0_N$and $U = V^c$. Since $U = V^c$, $V^c$is a neutrosophic feebly open sets and follows that$V$ is neutrosophic feebly closed set and $U \neq 0_N$implies $V \neq 1_N$. But this is a contradiction to the fact that X is neutrosophic feebly C_5-connected.

Conversely, let U be a both neutrosophic feebly open set and V is neutrosophic feebly closed set in X such that $U \neq 0_N, U \neq 1_N$.Now take $U^c = V$is a neutrosophic feebly open set and $U \neq 1_N$ which implies $U^c = V \neq 0_N$which is a contradiction. Hence X is neutrosophic feebly C_5-connected.

Theorem 3.14. A neutrosophic topological space $(X, \tau)$is neutrosophic feebly connected space if and only if there exists no non-zero neutrosophic feebly open set$U$ and Vin (X, τ), such that $U = V^c$.

Proof. Necessity: Let U and V be two neutrosophic feebly open set in (X, τ)such that $U \neq 0_N, V \neq 0_N$and $U = V^c$. Therefore $V^c$is a neutrosophic feebly closed set. Since $U \neq 0_N, V \neq 1_N$. This implies V is a proper neutrosophic subset which is both neutrosophic feebly open set and neutrosophic feebly closed setin X. Hence X is not a neutrosophic feebly connected space. But this is a contradiction to our hypothesis. Thus, there exist no non-zero neutrosophic feebly open sets U and Vin X, such that $U = V^c$.

Sufficiency:Let U be both neutrosophic feebly open and neutrosophic feebly closed X such that $U \neq 0_N, U \neq 1_N$. Now let $V = U^c$. Then V is a neutrosophic feebly open set and $V \neq 1_N$. This implies $U^c = V \neq 0_N$, which is a contradiction to our hypothesis. Therefore X is neutrosophic feebly connected space.

Theorem 3.15. A neutrosophic topological space $(X, \tau)$is neutrosophic feebly connected space if and only if there exists no non-zero neutrosophic subsets$U$ and Vin (X, τ), such that $U = V^c$, $V = (NFcl(U))^c$ and $U = (NFcl(V))^c$.

Proof. Necessity: Let U and V be two neutrosophic subsets in (X, τ)such that $U \neq 0_N, V \neq 0_N$ and $U = V^c$, $V = (NFcl(U))^c$and $U = (NFcl(V))^c$.Since $(NFcl(U))^c$and $(NFcl(V))^c$are neutrosophic feebly open setsin X, U and V are neutrosophic feebly open setin X. This implies $X$is not a neutrosophic feebly connected space, which is a contradiction. Therefore, there exists no non-zero neutrosophic feebly open set$U$ and Vin X, such that $U = V^c, V = (NFcl(U))^c$ and $U = (NFcl(V))^c$.

Sufficiency:Let U be both neutrosophic feebly open and neutrosophic feebly closed setin X such that $U \neq 0_N, U \neq 1_N$.Now by taking V= U^cwe obtain a contradiction to our hypothesis. Hence X is neutrosophic feebly connected space.

Definition 3.16. A neutrosophic topological space (X,τ) is neutrosophic feebly strongly connected, if there exists no nonempty neutrosophic closed sets A and B in X such that $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$or $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \leq 1_N, \gamma_A + \gamma_B \leq 1_N$.

In other words, a neutrosophic topological space $X$is neutrosophic feebly strongly connected, if there exists no nonempty neutrosophic feebly closed sets A and B in X such that $A \wedge B = 0_N$.

Theorem 3,17. An neutrosophic topological space $(X,\tau)$is neutrosophic feebly strongly connected, if there exists no nonempty neutrosophic feebly open sets A and Bin X, $A \neq 1_N \neq B$such that $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$.

Proof. Let A and B be neutrosophic feebly open sets in X such that $A \neq 1 \neq B$ and $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$. If we take $C = A^c$and $D = B^c$, then C and D become neutrosophic feebly closed sets in X and $C \neq 0_N \neq D, \gamma_C + \gamma_D = \mu_A + \mu_B \geq 1_N, \mu_C + \mu_D = \sigma_A + \sigma_B \geq 1_N, \sigma_C + \sigma_D = \gamma_A + \gamma_B \leq 1_N$, a contradiction.

Conversely, use a similar technique as above.

Example 3.18. Let $X = \{a,b\}$and $\tau = \{0_N,\ 1_N,\ A,\ B\}$ where $A = \{< x, (0.6, 0.6), (0.6, 0.6), (0.6, 0.9) >\}$,$B$= $\{< x$,(0.3, 0.1),(0.2, 0.6),(0.2, 0.3)>}. Then A and B are neutrosophic feebly open sets in X, also $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N,\ \gamma_A + \gamma_B \leq 1_N$. Hence X is neutrosophic feebly strongly connected.

Theorem 3.19. Let $f: (X,\tau) \to (Y,\sigma)$be a neutrosophic feebly irresolute surjection, Xbe a neutrosophic feebly strongly connected. Then $Y$is also neutrosophic feebly strongly connected.

Proof. Assume that $Y$is not neutrosophic feebly strongly connected, then there exists nonempty neutrosophic feebly closed sets U and V in Ysuch that $U \neq 0_N, V \neq 0_N$and $U \wedge V = 0_N$. Since $f$is neutrosophic feebly irresolute mapping, $A = f^{-1}(U) \neq 0_N$, $B = f^{-1}(V) \neq 0_N$, which are neutrosophic feebly closed sets in X. Since f is a surjection,$f^{-1}(U) \wedge f^{-1}(V) = f^{-1}(0_N) = 0_N$, which implies $A \wedge B = 0_N$. Thus X is not a neutrosophic feebly strongly connected, which is a contradiction to our hypothesis. Hence Y is neutrosophic feebly strongly connected.

Remark 3.20. Neutrosophic feebly strongly connected and neutrosophic feebly -connected are independent as shown by the following example.

Example 3.21. Let $X = \{a,b\}$ and $\tau = \{0_N,\ 1_N,\ A,\ B\}$ where $A = \{<x$,(0.6, 0.6),(0.6, 0.6), (0.2, 0.3)>},$B =$ $\{< x$, (0.3, 0.1), (0.2, 0.6), (0.6, 0.9)>}. Then A and B are neutrosophic feebly open sets in X, also $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$. Hence X is neutrosophic feebly strongly connected. But X is not neutrosophic feebly C_5-connected, since A is both neutrosophic feebly open and neutrosophic feebly closed in X.

Example 3.22. Let = $\{a,b\}$, τ = $\{0_N, 1_N, A, B, A \vee B, A \wedge B\}$ where

$$A = \{< x, (0.6, 0.5), (0.3, 0.4), (0.6, 0.2) >\}$$
$$B = \{< x, (0.5, 0.4), (0.2, 0.4), (0.2, 0.2) >\}$$

Hence X is neutrosophic feebly C_5-connected, but X is not neutrosophic feebly strongly connected. Since $A$and B are neutrosophic feebly open sets in X such that $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$.

IV. Neutrosophic Feebly Compact Spaces

In this section we introduce the concept neutrosophic feebly compact spaces using neutrosophic feebly open sets and study some of their basic properties.

Definition 4.1.A collection $\mathcal{B}$ of neutrosophic feebly open sets in X is called a neutrosophic feebly open cover of a subset B of X if $B \leq \vee\{U_\alpha : U_\alpha \in \mathcal{B}\}$.

Definition 4.2. A topological space X is said to be neutrosophic feebly compact if every neutrosophic feebly open cover of X has a finite subcover.

Definition 4.3. A subset A of a topological space X is said to be neutrosophic feebly compact relative to Xif every neutrosophic feebly open cover of X has a finite subcover.

Theorem 4.4. Every neutrosophic feebly compact space is neutrosophic compact.

Proof. Let X be neutrosophic feebly compact. Suppose X is not neutrosophic compact. Then there exists a neutrosophic open cover $\mathcal{B}$ of X has no finite subcover. Since every neutrosophic open set is neutrosophic feebly open, then we have neutrosophic feebly open cover $\mathcal{B}$of X, which has no finite subcover. This is a contradiction to $X$is neutrosophic feebly compact. Hence X is neutrosophic compact.

Theorem 4.5.A neutrosophic feebly closed subset of a neutrosophic feebly compact space X is neutrosophic feebly compact relative to X.

Proof. Let A be a neutrosophic feebly closed subset of a neutrosophic feebly compact space X. Then A^c is neutrosophic feebly open in X. Let $\mathcal{B}$=$\{A_i : i \in I\}$be a neutrosophic feebly open cover of A. Then $\mathcal{B} \vee \{A^c\}$ is a neutrosophic feebly open cover of X. Since X is neutrosophic feebly compact, it has a finite subcover say $\{P_1, P_2, ..., P_n, A^c\}$. Then $\{P_1, P_2, ..., P_n\}$ is a finite neutrosophic feebly open cover. Thus A is neutrosophic feebly compact relative to X.

Theorem 4.6. Let $f: X \to Y$ be neutrosophic feebly continuous surjection and X be neutrosophic feebly compact. Then Y is neutrosophic compact.

Proof. Let $f: X \to Y$be a neutrosophic feebly continuous surjection and X be neutrosophic feebly compact. Let $\{V_\alpha\}$ be a neutrosophic open cover for Y. Since f is neutrosophic feebly continuous, $\{f^{-1}(V_\alpha)\}$ is a neutrosophic feebly open cover of X. Since X is neutrosophic feebly compact, $\{f^{-1}(V_\alpha)\}$ contains a finite subcover, namely $\{f^{-1}(V_{\alpha_1}), f^{-1}(V_{\alpha_2}), ..., f^{-1}(V_{\alpha_n})\}$. Since f is surjection, $\{V_{\alpha_1}, V_{\alpha_2}, ..., V_{\alpha_n}\}$ is a finite subcover for Y. Thus Y is neutrosophic compact.

Theorem 4.7. Let $f: X \to Y$ be a neutrosophic feebly open function and Y be neutrosophic feebly compact. Then X is neutrosophic compact.

Proof. Let $f: X \to Y$be a neutrosophic feebly open function and Y be neutrosophic feebly compact. Let $\{V_\alpha\}$ be a neutrosophic open cover for X. Since f is neutrosophic feebly open, $\{f(V_\alpha)\}$ is a neutrosophic feebly open cover of Y. Since Y is neutrosophic feebly compact, $\{f(V_\alpha)\}$ contains a finite sub neutrosophic feebly cover, namely $\{f(V_{\alpha_1}), f(V_{\alpha_2}), \dots, f(V_{\alpha_n})\}$. Then $\{V_{\alpha_1}, V_{\alpha_2}, \dots, V_{\alpha_n}\}$ is a finite subcover for X. Thus X is neutrosophic compact.

Theorem 4.8. The image of a neutrosophic compact space under a neutrosophic continuous map is neutrosophic feebly compact.

Proof. Let $f: (X, \tau) \to (Y, \sigma)$be a neutrosophic feebly continuous map from a neutrosophic feebly compact space (X, τ)onto a feebly topological space (Y, σ). Let $\{A_i: i \in I\}$ be a neutrosophic open cover of (Y, σ). Since f is neutrosophic continuous, $\{f^{-1}(A_i): i \in I\}$ is a neutrosophic feebly open cover of (X, τ). As $(X, \tau)$is neutrosophic feebly compact, the neutrosophic open cover $\{f^{-1}(A_i): i \in I\}$ of (X, τ)has a finite subcover$\{f^{-1}(A_i): i = 1,2,3, \dots, n\}$. Therefore $U = \cup_{i \in I} f^{-1}(A_i)$. Then $f(U) = \cup_{i \in I} A_i$, that is $V = \cup_{i \in I} A_i$. Thus $\{A_1, A_2, \dots A_n\}$ is a finite subcover of $\{A_i: i \in I\}$ for (Y, σ). Hence $(Y, \sigma)$is neutrosophic feebly compact.

V. Conclusion

Neutrosophic feebly connectedness and neutrosophic feebly disconnectedness in the neutrosophic topological space have been introduced. Also, we have introduced the neutrosophic feebly compactness and we hope that the findings in this chapter will help researchers to enhance and promote the further study on neutrosophic topology to carry out general framework for their applications in practical life.

References

[1]. Albowi, SA, Salama, A, and Mohamed Eisa, 2013, New Concepts of Neutrosophic Sets, *International Journal of Mathematics and Computer Applications Research*, Vol.3(3), pp. 95-102.

[2]. Atanassov, K, 1986, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems,* 20, pp. 87-96.

[3]. Chang, CL, 1968, Fuzzy Topological Spaces, *J. Math. Anal. Appl.* 24, pp.182-190.

[4]. Dogan Coker, 1997, An Introduction to Intuitionistic Fuzzy Topological Spaces, *Fuzzy Sets and Systems*, 88, pp. 81-89.

[5]. FloretinSmaradache, 2010, Neutrosophic Set:-A Generalization of Intuitionistic Fuzzy Set, *Journal of Defense Resources Management.* 1, pp. 107-116.

[6]. FloretinSmarandache, 2005,Neutrosophic set - A Generalization of the Intuitionistic Fuzzy Set, *International Journal of Pure and Applied Mathematics,* 24(3), pp. 287-297.

[7]. FloretinSmarandache, 2002, Neutrosophy and Neutrosophic Logic, *First International Conference on Neutrosophy, Neutrosophic Logic, set, Probability, and statistics, University of New Mexico, Gallup*, NM 87301. USA

[8]. FloretinSmarandache, 1999, A Unifying Field in Logics: Neutrosophic Logic,Neutrosophy, Neutrosophic Set, Neutrosophic Probability, *American Research Press*.

[9]. Iswarya, P, and Bageerathi, K, 2016, On Neutrosophic Semi-Open Sets in Neutrosophic Topological Spaces, *International Journal of Mathematics Trends and Technology (IJMTT)*,Vol 37(3), pp. 24-33.

[10]. JeyaPuvaneswari, P, Bageerathi, K, 2017, On Neutrosophic Feebly open sets in Neutrosophic topological spaces, *International Journal of Mathematics Trends and Technology,* Vol. 41(3), pp. 230 – 237.

[11]. JeyaPuvaneswari, P, Bageerathi, K, 2019, On Neutrosophic Feebly Continuous Functions *Journal of Emerging Technologies and Innovative Research,* Vol. 6 (2) (ISSN No. 2349 – 5162) (UGC Approved Journal No. 63975)

[12]. JeyaPuvaneswari, P, Bageerathi, K, 2019, Some Functions Concerning Neutrosophic Feebly Open & Closed Sets. (Submitted)

[13]. Salama, AA, Smarandache, Floretin, and Valeri, K, 2014,Neutrosophic Closed Set and Neutrosophic Continuous Functions, *Neutrosophic Sets and Systems,*4, pp. 4-8.

[14]. Salama, AA, and Alblowi, SA, 2012, Generalized Neutrosophic Set and Generalized Neutrousophic Topological Spaces, *Journal computer Sci. Engineering,* Vol. (2) No. 7.

[15]. Salama, AA, Alblowi, AA, 2012, Neutrosophic Set and Neutrosophic Topological Space, *ISOR Journal of Mathematics*, Vol. (3), Issue (4), pp. 31-35.

[16]. Turnali, N and Coker,D, 2000, Fuzzy connectedness in intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 116, 369-375.

[17]. Zadeh, LA, 1965, Fuzzy Sets, *Inform and Control*, Vol. 8, pp. 315-353.